Alternative Formulation of The Quantum Electroweak Theory

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The quantization of the electroweak theory is performed starting from the Lagrangian given in the so-called unitary gauge in which the unphysical Goldstone fields disappear. In such a Lagrangian, the unphysical longitudinal components of the gauge fields and the residual gauge degrees of freedom are naturally eliminated by introducing the Lorentz gauge condition and the ghost equation. In this way, the quantum theory given in α-gauge is perfectly established in the Lagangian formalism by the Faddeev-Popov approach or the Lagrange multiplier method in the framework of SU(2)×U(1) gauge symmetry. The theory established is not only simpler than the ordinary R_α−gauge theory, but also explicitly renormalizable. The unitarity of the S-matrix is ensured by the α−limiting procedure proposed previously. Especially, it is shown that the electroweak theory without involving the Higgs boson can equally be formulated whitin the $SU(2) \times U(1)$ symmetry and exhibits good renormalizability. The unitarity of such a theory may also be guaranteed by the α-limiting procedure.

PACS: 11.15-qy, 11.10. Gh; 12.20.-m

Keywords: Electroweak theory, unitary gauge, quantization.

1. INTRODUCTION

In our recent publication[1], it has been shown that the conventional viewpoint that the massive gauge field theory can not be set up without introducing the Higgs mechanism is not true. In fact, a certain massive gauge field theories can be well established on the basis of gauge-invariance principle without recourse to the Higgs mechanism. The essential viewpoints to achieve this conclusion are: (a) a massive gauge field must be viewed as a constrained system in the whole space of vector potentials and the Lorentz condition, as a necessary constraint, must be introduced from the beginning and imposed on the Lagrangian; (b) The gauge-invariance of a gauge field theory should be generally examined from the action of the field other than from the Lagrangian because the action is of more fundamental dynamical meaning than the Lagrangian. Particularly, for a constrained system such as the massive gauge field, the gauge-invariance should be seen from its action given in the physical space defined by the Lorentz condition. This concept is well-known in Mechanics; (c) In the physical space, only infinitesimal gauge transformations are possibly allowed and necessary to be considered. This fact was clarified originally in Ref.[2]. Based on these points of view, it is easy to see that the massive gauge field theory in which all the gauge bosons have the same masses are surely gauge-invariant. Obviously, the Quantum chromodynamics (QCD) with massive gluons fulfils this requirement because all the gluons can be considered to have the same masses. As has been proved, such a QCD is not only renormalizable, but also unitary[3]. The renormalizability and unitarity are warranted by the fact that the unphysical degrees of freedom existing in the massive Yang-Mills Lagrangian are completely eliminated by the introduced constraint conditions on the gauge field and the gauge group, i.e. the Lorentz condition and the ghost equation. In the tree-diagram level, as easily verified, the unphysical longitudinal part of the gauge boson propagator gives a vanishing contribution to the S-matrix elements because in the QCD Lagrangian, there only appear the vector currents of quarks coupled to the gluon fields and in each current, the quarks and /or the antiquarks are of the same flavor. For loop-diagrams, it has been proved that the unphysical intermediate states are all cancelled out in S-matrix elements. However, for the weak interaction, apart from the vector currents of fermions, there appear the axial vector currents of fermions. In particular, the charged and neutral gauge bosons as well as the charged and neutral fermions are required to have different masses. In this case, it is impossible to construct a gauge-invariant action without introducing the Higgs mechanism. Within the framework of the Higgs mechanism, the electro-weak-unified theory was successfully set up on the basis of the $SU(2) \times U(1)$ gauge-invariance[4−6]. The physical implication of such a theory was originally revealed in the so-called unitary gauge in which all the Goldstone fields are absent. The Lagrangian given in the unitary gauge is obtained from the original $SU(2) \times U(1)$ gauge-symmetric Lagrangian which includes all the scalar fields in it by the Higgs transformation.[4−6]. Such a Lagrangian was initially used to establish the quantum theory. The free massive gauge boson propagator derived from the this theory is of the form[4−8]

$$iD^j_{\mu\nu}(k) = \frac{-i}{k^2 - M_j^2 + i\varepsilon}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_j^2}\right) \tag{1.1}$$

where $j = W^{\pm}$ or Z^0. It is the prevailing point of view that the above propagator explicitly ensures the unitarity of the S-matrix because except for the physical pole at $k^2 = M_j^2$, there are no other unphysical poles to appear in the propagator. However, due to the bad ultraviolet divergence of the second term in the propagator shown in Eq.(1.1), as pointed out in the literature[7], Green's functions defined in the unitary gauge theory are unrenormalizable. The uinrenormalizability of the unitary gauge theory arises from the fact that the unphysical degrees of freedom, i. e. the (four-dimensionally) longitudinal gauge fields and the residual gauge degrees of freedom contained in the Lagrangian given in the unitary gauge are not eliminated by introducing appropriate constraint conditions. Later, the quantization of the electroweak theory was elegantly carried out in the so-called R_α- gauge by the Faddeev-Popov approach[7,8]. In this quantization, the authors started from the original Lagrangian which contains all the Goldstone fields in it and introduced the R_α-gauge conditions[7,8]

$$\partial^\mu A_\mu^a + \frac{i}{2}\alpha g(\phi^+ \tau^a V - V^+ \tau^a \phi) = 0 \tag{1.2}$$

$$\partial^\mu B_\mu + \frac{i}{2}\alpha g'(\phi^+ V - V^+ \phi) = 0 \tag{1.3}$$

where A_μ^a and B_μ are the $SU(2)_T$ and $U(1)_Y$ gauge fields, g and g' are the $SU(2)_T$ and $U(1)_Y$ coupling constants, respectively, α is the gauge parameter, τ^a are the Pauli matrices,

$$\phi = \phi' + V \tag{1.4}$$

here

$$\phi' = \frac{1}{\sqrt{2}} \begin{pmatrix} G_1 + iG_2 \\ H + iG_0 \end{pmatrix} \tag{1.5}$$

in which G_1, G_2 and G_0 are the Goldstone fields and H is the Higgs field and

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{1.6}$$

is the vacuum state doublet in which v represents the vacuum expectation value of the scalar field. The quantum theory built in the R_α gauge has been widely accepted because the massive gauge boson propagator given in this gauge is of the form

$$iD_{\mu\nu}^\alpha(k) = -i \left\{ \frac{g_{\mu\nu} - k_\mu k_\nu/k^2}{k^2 - M_k^2 + i\varepsilon} + \frac{\alpha k_\mu k_\nu/k^2}{k^2 - \alpha M_k^2 + i\varepsilon} \right\} \tag{1.7}$$

With taking different values of the gauge parameter, we have different propagators such as the ones given in the Landau gauge ($\alpha = 0$), the 't Hooft-Feynman gauge ($\alpha = 1$) and the unitary gauge ($\alpha \to \infty$), respectively. Since the above propagator shows good ultraviolet behavior and therefore satisfies the power counting argument of renormalizability, the quantum theory formulated in the R_α - gauge is considered to be renormalizable and there were some formal proofs presented in the previous literature which seem to assert this point[8−15]. Recently, however, H. Cheng and S. P. Li have presented a strong argument which indicates that the quantum electroweak theory given in the R_α-gauge is difficult to be renormalized, particularly, the occurrence of double poles which are ultraviolet divergent renders the multiplicative renormalization of the propagators to be impossible [16].Obviously, the difficulty of the renormalization of the R_α-gauge theory originates from the fact that the unphysical degrees of freedom contained in the Lagrangian which was chosen to be the starting point of quantization are not completely eliminated by the introduced R_α- gauge conditions. This point may clearly be seen from the Landau gauge in which the R_α- gauge conditions are reduced to the Lorentz gauge conditions

$$\partial^\mu A_\mu^a = 0 \tag{1.8}$$

$$\partial^\mu B_\mu = 0 \tag{1.9}$$

These conditions imply vanishing of the longitudinal fields, $A_{L\mu} = 0$ and $B_{L\mu} = 0$. But, the unphysical Goldstone fields could not be constrained by the above constraint conditions. They are still remained in the Lagrangian and play an important role in perturbative calculations (see the illustration in Appendix A).

In this paper, we attempt to propose an alternative formulation of the quantum electroweak theory in which the unphysical Goldstone bosons and even the Higgs particle do not appear. According to the general principle of constructing a renormalizable quantum field theory for a constrained system such as the massive and massless gauge fields, the unphysical degrees of freedom appearing in the Lagrangian ought to be all eliminated by introducing necessary constraint conditions[1,2]. This suggests that the quantization of the electroweak theory may suitably be performed starting from the Lagrangian given in the unitary gauge[4,5]. This Lagrangian originally was considered to be physical because the unphysical Goldstone fields disappear in it. However, in such a Lagrangian still exist the longitudinal components of the gauge fields and the residual gauge degrees of freedom. These unphysical degrees of freedom may completely be removed by introducing the Lorentz gauge conditions shown in Eqs.(1.8) and (1.9) and the constraint condition on the gauge group (the ghost equation). In this way, the quantum electroweak theory given in α-gauge may perfectly be set up by applying the Faddeev-Popov approach[2] or the Lagrange undetermined multiplier method[1]. In such a quantum theory, the massive gauge boson propagators are still of the form as denoted in Eq.(1.7) and hence exhibit explicit renormalizability of the theory.

There are two questions one may ask: One is that what is the gauge symmetry of the theory established from the Lagrangian given in the unitary gauge ? Another is how to ensure the unitarity of S-matrix elements evaluated from such a theory? For the first question, we would like to mention that ordinarily, the Lagrangian given in the unitary gauge, which is obtained from the original $SU(2) \times U(1)$ symmetric Lagrangian by the Higgs transformation, is explained to describe a spontaneously symmetry-broken theory for which only the electric charge U(1)-symmetry is remained. This concept comes from the assumption that the vacuum state in Eq.(1.6) is invariant under gauge transformations. However, this concept is in contradiction with the viewpoint adopted in the widely accepted quantum theory established in Refs.[6-7] that the vacuum state in (1.6) is not gauge-invariant, it undergoes the same gauge transformations as the scalar field ϕ' shown in Eq.(1.5) does. Therefore, the present quantum electroweak theory given in the R_α gauge quantization is set up from beginning to end on the basis of $SU(2) \times U(1)$ gauge-symmetry and, as pointed out by t'Hooft[17], there actually is no spontaneous symmetry-breaking to appear in such a theory. The viewpoint mentioned above is essential. It enables us to build up a correct electroweak theory from the Lagrangian given in the unitary gauge which is still of the $SU(2) \times U(1)$ gauge-symmetry. Otherwise, one could not build a reasonable quantum theory from the unitary gauge Lagrangian for the electroweak-interacting system. To answer the second question, it is noted that the propagator in Eq.(1.1) is derived from the on mass-shell vector potential, while the propagator in Eq.(1.7) is derived from the off mass-shell vector potential (see the explanation in the Appendix B). The latter propagator may be derived from the generating functional of Green's functions which is given In the path-integral quantization. This propagator is suitable for the renormalization of off-shell Green's functions and also for the renormalization of on-shell S-matrix elements because the propagators appearing in the loop diagrams of S-matrix elements are off-mass-shell even though the external momenta of the S-matrix elements are on the mass-shell. Clearly, the propagators in Eq.(1.7) which is given in the α-gauge can be viewed as a parametrization of the propagators given in the unitary gauge since in the limit: $\alpha \to \infty$, the former propagator is converted to the latter one. Therefore, calculation of a physical quantity may safely be done in the α-gauge and then, as was proposed and demonstrated previously[18,19], the $\alpha-$limiting procedure is necessary to be taken in the final step of the calculation of a S-matrix element.

Furthermore, we try to build a theory in which the Higgs boson is removed. This can be done by the requirement that the scalar field ϕ defined in Eq.(1.4), which is a vector in the four-dimensional functional space, is limited to the subspace in which the magnitude of the vector is equal to its vacuum expectation value v. Since the vacuum state in Eq.(1.5) obeys the gauge-transformation law as the same as the scalar field ϕ does, it will be shown that the action without involving the Higgs boson in it is still of $SU(2) \times U(1)$ gauge-symmetry under the introduced Lorentz condition. Therefore, the quantum electroweak theory without involving the Higgs boson may also be set up within the framework of $SU(2) \times U(1)$ gauge-symmetry. The gauge boson propagators derived from such a theory is still represented in Eq.(1.7). Therefore, the renormalizability of such a theory is no problems and its unitarity can also be guaranteed by the α-limiting procedure.

The rest of this paper is arranged as follows. In Sect.2, we describe the quantization based on the Lagrangian given in the unitary gauge and the Lorentz gauge condition. In Sect.3, some Ward-Takahashi identities[20] are derived. Sect.4 serves to inclusion of the quarks. Sect.4 is used to formulate the theory without involving the Higgs boson. In the last section, some remarks are presented. In Appendix A, we take an example to illustrate the role of Goldstone particles in the ordinary R_α-gauge theory. Appendix B is used to explain the difference and the relation between the both propagators derived in the unitary gauge and the α-gauge.

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2. QUANTIZATION

In this section, we describe the quantization of the electroweak theory based on the Lagrangian given in the unitary gauge. For one generation of leptons , the Lagrangian is[4,5]

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_f + \mathcal{L}_\phi \tag{2.1}$$

where $\mathcal{L}_g, \mathcal{L}_f$ and $\mathcal{L}_\phi$ are the parts of the Lagrangian for the gauge fields, the lepton fields and the scalar fields, respectively. They are written in the following

$$\mathcal{L}_g = -\frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \tag{2.2}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu \tag{2.3}$$

and

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{2.4}$$

$$\mathcal{L}_f = \overline{L} i\gamma^\mu D_\mu L + \overline{l}_R i\gamma^\mu D_\mu l_R \tag{2.5}$$

where

$$L = \begin{pmatrix} \nu_L \\ l_L \end{pmatrix} \tag{2.6}$$

is the doublet formed by a left-handed neutrino field ν_L and a left-handed charged lepton field l_l, l_R is the singlet of a right-handed charged lepton field and

$$D_\mu = \partial_\mu - ig\frac{\tau^a}{2} A^a_\mu - ig'\frac{Y}{2} B_\mu \tag{2.7}$$

is the covariant derivative in which $\frac{\tau^a}{2}(a=1,2,3)$ and $\frac{Y}{2}$ are the generators of $\mathrm{SU(2)}_T$ and $\mathrm{U(1)}_Y$ groups respectively.

$$\mathcal{L}_\phi = (D^\mu \phi_0)^+ (D_\mu \phi_0) - \mu^2 \phi_0^+ \phi_0 - \lambda(\phi_0^+ \phi_0)^2 - f_l(\overline{L}\phi_0 l_R + \overline{l}_R \phi_0^+ L) \tag{2.8}$$

where

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ H+v \end{pmatrix} \tag{2.9}$$

in which $v = \sqrt{-2\mu^2/\lambda}$. The ϕ_0 is a special configuration of the scalar fields which is connected with the field configuration shown in Eq.(1.4) by a gauge transformation $\phi = U\phi_0$ where $U = \exp\{\frac{i}{2}(g\tau^a \theta^a + g'\theta^0)\}$.

As mentioned in the Introduction, in the Lagrangian written above still exist the unphysical longitudinal parts of the gauge fields which are necessary to be eliminated by introducing the Lorentz gauge conditions shown in Eqs.(1.7) and (1.8). The necessity of introducing the Lorentz condition in this case may also be seen from the R_α-gauge condition. In fact, considering that the conditions in Eqs.(1.2) and (1.3) should suit to any field configuration, certainly, it is suitable for the field configuration given in the unitary gauge. It is easy to verify that

$$\phi_0^+ \tau^a V - V^+ \tau^a \phi_0 = 0 \tag{2.10}$$

$$\phi_0^+ V - V^+ \phi_0 = 0 \tag{2.11}$$

so that in the unitary gauge, the R_α-gauge conditions is reduced to the Lorentz gauge conditions which are now rewritten as

$$\partial^\mu A^i_\mu = 0 \tag{2.12}$$

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where we have set $A_\mu^0 \equiv B_\mu$ and let $i = 0, 1, 2, 3$.

Before performing the quantization of the electroweak theory starting from the Lagrangian and the Lorentz condition described above by the Faddeev-Popov method[2], it is at first stressed that the Lagrangian $\mathcal{L}(x)$ described above, as mentioned in Introduction, still has the $SU(2) \times U(1)$ gauge symmetry, unlike the ordinary concept that the Lagrangian merely has the electric charge $U(1)$-symmetry. For the Lagrangians $\mathcal{L}_g$ and $\mathcal{L}_f$, it is clear that they are still $SU(2) \times U(1)$ gauge-symmetric in the unitary gauge. While, for the Lagrangian $\mathcal{L}_\phi$, as can easily be verified, it also keeps invariant under the following $SU(2) \times U(1)$ gauge transformations:

$$\delta\phi_0 = \frac{i}{2}(g\tau^a\theta^a + g'\theta^0)\phi_0 \tag{2.13}$$

$$\delta L = \frac{i}{2}(g\tau^a\theta^a - g'\theta^0)L \tag{2.14}$$

$$\delta l_R = -ig'\theta^0 l_R \tag{2.15}$$

$$\delta A_\mu^i = D_\mu^{ij}\theta^j \tag{2.16}$$

where

$$D_\mu^{ij} = \delta^{ij}\partial_\mu - g\varepsilon^{ijk}A_\mu^k \tag{2.17}$$

In the above, the eigen-equations

$$YL = -L, Yl_R = -2l_R, Y\phi_0 = \phi_0, \tau^a l_R = 0 \tag{2.18}$$

and the definition

$$\varepsilon^{ijk} = \{ \begin{matrix} \epsilon^{abc}, \text{if } i, j, k = a, b, c = 1, 2, 3; \\ 0, \text{if } i, j \text{ and/or } k = 0 \end{matrix} \tag{2.19}$$

have been used. Here, as said in Introduction, we adopt the concept implied in the previous quantization of the electroweak theory carried out in the R_α-gauge[7,8] that the vacuum state shown in Eq.(1.6) is not set to be gauge-invariant under the $SU(2) \times U(1)$ gauge transformations. This vacuum state as well as the field ϕ_0 shown in Eq.(2.9) undergo the same gauge-transformation as the original scalar field ϕ denoted in Eq.(1.4) so that either the original Lagrangian $\mathcal{L}_\phi$ which includes the Goldstone fields in it or the Lagrangian $\mathcal{L}_\phi$ shown in Eq.(2.8) is still of the $SU(2) \times U(1)$ gauge symmetry. Next, it is pointed out that to obtain a proper form of the ghost field Lagrangian in the general α-gauge, it is necessary to add the identities in Eqs.(2.10) and (2.11) to the Lorentz condition in Eq.(2.12) and write the constraint condition in a generalized form

$$F^i[A, \phi_0] + \alpha\lambda^i = 0 \tag{2.20}$$

where λ^i is an auxiliary function and

$$F^i[A, \phi_0] = \partial^\mu A_\mu^i + \frac{i}{2}\alpha g^i(\phi_0^+\tau^i V - V^+\tau^i\phi_0) \tag{2.21}$$

here we have set $\tau^0 = 1$ and $g^i = g$, if $i = 1, 2, 3$ and $g^i = g'$, if $i = 0$. This is because for the quantization performed in the Lagrangian formalism, one has to make gauge transformations to the gauge condition which will connect the Higgs field to other scalar fields.

Now we are in a position to carry out the quantization starting from the Lagrangian given in the unitary gauge and the Lorentz condition. According to the general procedure of the Faddeev-Popov approach of quantization[2,21,22]. we insert the following identity

$$\Delta[A, \phi_0] \int D(g)\delta[F[A^g, \phi_0^g] + \alpha\lambda] = 1 \tag{2.22}$$

, where g is an element of the $SU(2) \times U(1)$ group, into the vacuum-to-vacuum transition amplitude, obtaining

$$Z[0] = \frac{1}{N} \int D\left(\Psi\right) D(g) \Delta[A, \phi_0] \delta[F[A^g, \phi_0^g] + \alpha\lambda] e^{i \int d^4x \mathcal{L}(x)} \tag{2.23}$$

where $\mathcal{L}(x)$ is the Lagrangian denoted in Eqs.(2.1)-(2.9) and Ψ stands for all the field variables $(\bar{l}, l, \bar{v}, v, A_\mu^a, B_\mu, H)$ in the Lagrangian. Let us make a gauge transformation: $A_\mu^i \to (A^{g^{-1}})_\mu^i$ and $\phi_0 \to \phi_0^{g^{-1}}$ to the functional in Eq.(2.23). Since the Lagrangian $\mathcal{L}(x)$ is gauge-invariant and the functional $\Delta[A, \phi_0]$ as well as the integration measure, as already proved in the literature[2,8,15], are all gauge-invariant, the integral over the gauge group, as a constant, may be factored out from the integral over the fields and put in the normalization constant N. Thus, we have

$$Z[0] = \frac{1}{N} \int D(\Psi) \Delta[A, \phi_0] \delta[F[A, \phi_0] + \alpha\lambda] e^{i \int d^4 \chi \mathcal{L}(x)} \tag{2.24}$$

The functional $\Delta[A, \phi_0]$ in the above, which may be evaluated from the identity in Eq.(2.22) and the gauge-transformation shown in Eqs.(2.13)-(2.17), will be expressed as[2,21]

$$\Delta[A, \phi_0] = \det M[A, \phi_0] \tag{2.25}$$

where $M[A, \phi_0]$ is a matrix whose elements are

$$M^{ij}(x, y) = \frac{\delta F_\theta^i(x)}{\delta\theta^j(y)} |_{\theta=0} = \partial_x^\mu [D_\mu^{ij}(x)\delta^4(x-y)] + \frac{i}{8}\alpha g^i g^j [V^+ \tau^i \tau^j \phi_0(x) + \phi_0^+(x)\tau^j \tau^i V]\delta^4(x-y) \tag{2.26}$$

Employing the familiar representation for the determinant[2]

$$\det M = \int D(\overline{C}, C) e^{i \int d^4x d^4y \overline{C}^i(x) M^{ij}(x,y) C^j(y)} \tag{2.27}$$

where $\overline{C}^i$ and C^i are the mutually conjugate ghost field variables, integrating Eq.(2.24) over the functions $\lambda^i(x)$ with the weight $exp[-\frac{i}{2}\alpha(\lambda^i)^2]$ and then introducing the external source terms for all the fields, we obtain from Eq.(2.24) the generating functional of Green's functions such that

$$Z[J] = \frac{1}{N} \int D(\Phi) e^{i \int d^4x [\mathcal{L}_{eff} + J \cdot \Phi]} \tag{2.28}$$

where Φ and J designate respectively all the fields and external sources including the ghosts and $\mathcal{L}_{eff}$ is the effective Lagrangian for the system under consideration. With the following definitions of the field variables

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp iA_\mu^2) \tag{2.29}$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} A_\mu^3 \\ B_\mu \end{pmatrix} \tag{2.30}$$

$$C^\pm = \frac{1}{\sqrt{2}}(C^1 \mp iC^2), \overline{C}^\pm = \frac{1}{\sqrt{2}}(\overline{C}^1 \mp i\overline{C}^2) \tag{2.31}$$

and

$$\begin{pmatrix} C_z \\ C_\gamma \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} C^3 \\ C^0 \end{pmatrix} \tag{2.32}$$

$$\begin{pmatrix} \overline{C}_z \\ \overline{C}_\gamma \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} \overline{C}^3 \\ \overline{C}^0 \end{pmatrix} \tag{2.33}$$

where θ_w is the Weinberg angle, the effective Lagrangian will be represented as

$$\mathcal{L}_{eff} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_H + \mathcal{L}_{gf} + \mathcal{L}_{gh} \tag{2.34}$$

where

$$\mathcal{L}_G = -\tfrac{1}{2}W^+_{\mu\nu}W^{-\mu\nu} - \tfrac{1}{4}(Z^{\mu\nu}Z_{\mu\nu} + A^{\mu\nu}A_{\mu\nu}) + M_w^2 W^+_\mu W^{-\mu} + \tfrac{1}{2}M_z^2 Z^\mu Z_\mu$$
$$+ig[(W^+_{\mu\nu}W^{-\mu} - W^-_{\mu\nu}W^{+\mu})(\sin\theta_w A^\nu + \cos\theta_w Z^\nu) + W^+_\mu W^-_\nu(\sin\theta_w A^{\mu\nu} + \cos\theta_w Z^{\mu\nu})]$$
$$+g^2\{W^+_\mu W^-_\nu(\sin\theta_w A^\mu + \cos\theta_w Z^\mu)(\sin\theta_w A^\nu + \cos\theta_w Z^\nu) - W^+_\mu W^{-\mu}(\sin\theta_w A_\nu + \cos\theta_w Z_\nu)^2$$
$$+\tfrac{1}{2}[(W^+_\mu)^2(W^-_\nu)^2 - (W^+_\mu W^{-\mu})^2]\} \tag{2.35}$$

in which

$$W^\pm_{\mu\nu} = \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu, Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu, A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2.36}$$

and

$$M_w = \frac{1}{2}gv, M_z = \frac{M_w}{\cos\theta_w} \tag{2.37}$$

$$\mathcal{L}_F = \overline{\nu}i\gamma^\mu\frac{1}{2}(1-\gamma_5)\partial_\mu\nu + \overline{l}(i\gamma^\mu\partial_\mu - m_l)l + \frac{g}{\sqrt{2}}(j^-_\mu W^{+\mu} + j^+_\mu W^{-\mu}) - ej^{em}_\mu A^\mu + \frac{e}{\sin 2\theta_w}j^0_\mu Z^\mu \tag{2.38}$$

in which

$$j^-_\mu = \overline{\nu}\gamma^\mu\frac{1}{2}(1-\gamma_5)l = (j^+_\mu)^+ \tag{2.39}$$

$$j^{em}_\mu = \overline{l}\gamma_\mu l \tag{2.40}$$

$$j^0_\mu = \overline{\nu}\gamma_\mu\frac{1}{2}(1-\gamma_5)\nu - \overline{l}\gamma_\mu\frac{1}{2}(1-\gamma_5)l + 2\sin^2\theta_w j^{em}_\mu \tag{2.41}$$

and

$$m_l = \frac{1}{\sqrt{2}}f_l v \tag{2.42}$$

$$\mathcal{L}_H = \frac{1}{2}(\partial^\mu H)^2 - \frac{1}{2}m_H^2 H^2 + \frac{g}{4}(W^{+\mu}W^-_\mu + \frac{1}{2\cos\theta_w}Z^\mu Z_\mu)(H^2 + 2vH) - \lambda v H^3 - \frac{\lambda}{4}H^4 - \frac{f_l}{\sqrt{2}}\overline{l}lH \tag{2.43}$$

$$\mathcal{L}_{gf} = -\frac{1}{\alpha}[\partial^\mu W^+_\mu \partial^\nu W^-_\nu + \frac{1}{2}(\partial^\mu Z_\mu)^2 + \frac{1}{2}(\partial^\mu A_\mu)^2] \tag{2.44}$$

and

$$\mathcal{L}_{gh} = \overline{C}^-(\Box + \alpha M_w^2)C^+ + \overline{C}^+(\Box + \alpha M_w^2)C^- + \overline{C}_z(\Box + \alpha M_z^2)C_z + \overline{C}_\gamma\Box C\gamma - ig\{(\partial^\mu\overline{C}^+ C^-$$
$$-\partial^\mu\overline{C}^- C^+)(\cos\theta_w Z_\mu + \sin\theta_w A_\mu) + (\partial^\mu\overline{C}^- W^+_\mu - \partial^\mu\overline{C}^+ W^-_\mu)(\cos\theta_w C_z + \sin\theta_w C_\gamma)$$
$$+(\cos\theta_w\partial^\mu\overline{C}_z + \sin\theta_w\partial^\mu\overline{C}_\gamma)(C^+ W^-_\mu - C^- W^+_\mu)\} + \tfrac{1}{2}\alpha gM_w H(\overline{C}^+ C^- + \overline{C}^- C^+ + \frac{1}{\cos^2\theta_w}\overline{C}_z C_z) \tag{2.45}$$

The external source terms in Eq.(2.28) are defined by

$$J\cdot\Psi = J^{-\mu}W^+_\mu + J^+_\mu W^{-\mu} + J^z_\mu Z^\mu + J^\gamma_\mu A^\mu + JH + \overline{\xi}_l l + \overline{l}\xi_l + \overline{\xi}_\nu\nu + \overline{\nu}\xi_\nu$$
$$+\overline{\eta}^+ C^- + \overline{\eta}^- C^+ + \overline{C}^+\eta^- + \overline{C}^-\eta^+ + \overline{\eta}_z C_z + \overline{C}_z\eta_z + \overline{\eta}_\gamma C_\gamma + \overline{C}_\gamma\eta_\gamma \tag{2.46}$$

For the case of three generations of leptons, in Eq.(2.38) there will be the kinetic energy terms for three neutrinos and three leptons, and the sums over the number of generations of leptons should be included in Eqs.(2.39)-(2.41). Correspondingly, the external source terms will be extended to the case for three generations of leptons.

It is interesting to note that the quantized result shown above can be obtained more directly by the Lagrange multiplier method[1]. By this method, we may incorporate the constraint condition in Eq.(2.20) into the Lagrangian in Eq.(2.1) which is now extended to the form

$$\mathcal{L}' = \mathcal{L} - \frac{1}{2}\alpha(\lambda^i)^2 \tag{2.47}$$

where $\mathcal{L}$ was written in Eqs.(2.1)-(2.9) and thus obtain a generalized Lagrangian

$$\begin{aligned}\mathcal{L}_\lambda &= \mathcal{L}' + \lambda^i F^i[A, \phi_0] + \alpha(\lambda^i)^2 \\ &= \mathcal{L} + \lambda^i F^i[A, \phi_0] + \tfrac{1}{2}\alpha(\lambda^i)^2\end{aligned} \tag{2.48}$$

To construct a gauge-invariant theory, it is necessary to require the action given by the above Lagrangian to be gauge-invariant under the gauge transformations in Eqs.(2.13)-(2.16) and the constraint condition in Eq.(2.20)

$$\delta S_\lambda = \int d^4x\, \delta\mathcal{L}_\lambda = \int d^4x\, \lambda^i(x)\{\partial_x^\mu[D_\mu^{ij}(x)\theta^j(x)] + \frac{i}{8}\alpha g^i g^j[V^+\tau^i\tau^j\phi_0(x) + \phi_0^+(x)\tau^j\tau^i V]\theta^j(x)\} = 0 \tag{2.49}$$

Since $\lambda^i(x) \neq 0$, we have

$$\partial_x^\mu[D_\mu^{ij}(x)\theta^j] + \frac{i}{8}\alpha g^i g^j[V^+\tau^i\tau^j\phi_0(x) + \phi_0^+(x)\tau^j\tau^i V]\theta^j = 0 \tag{2.50}$$

This just is the constraint equations on the gauge group. Setting $\theta^\alpha = \xi C^\alpha$ where ξ is an anticommuting number and C^α are the ghost fields, we obtain from Eq.(2.50) the ghost equations

$$\partial_x^\mu[D_\mu^{ij}(x)C^j] + \frac{i}{8}\alpha g^i g^j[V^+\tau^i\tau^j\phi_0(x) + \phi_0^+(x)\tau^j\tau^i V]C^j = 0 \tag{2.51}$$

Since these equations are the alternatives of the constraints on the gauge group, they may also be incorporated into the Lagrangian $\mathcal{L}_\lambda$ by the Lagrange multiplier method. Thus, we have

$$\begin{aligned}\mathcal{L}_\lambda^* &= \mathcal{L} + \lambda^i F^i[A.\phi_0] + \tfrac{1}{2}\alpha(\lambda^i)^2 + \overline{C}^i\{\partial_x^\mu[D_\mu^{ij}(x)C^j] \\ &+ \tfrac{i}{8}\alpha g^i g^j[V^+\tau^i\tau^j\phi_0(x) + \phi_0^+(x)\tau^j\tau^i V]C^j\}\end{aligned} \tag{2.52}$$

where $\overline{C}^i$, acting as the Lagrange multipliers, are another kind of ghost field variables which are conjugate to the variables C^j.

As we learn from the Lagrange multiplier method, all the variables in the Lagrangian $\mathcal{L}_\lambda^*$, including the dynamical variables, the constrained variables and the Lagrange multipliers act as independent, varying arbitrarily. Therefore, we may directly utilize the Lagrangian $\mathcal{L}_\lambda^*$ to construct the generating functional of Green's functions so as to achieve the final goal of quantization

$$Z[J] = \frac{1}{N}\int D(\Psi)\,D(\lambda)\exp\{i\int d^4x(\mathcal{L}_\lambda^* + J\cdot\Psi)\} \tag{2.53}$$

where Ψ stands for all the field variables but the Lagrange multipliers and J designates all the corresponding external sources. On calculating the integral over λ^i, we precisely obtain the result as shown in Eqs.(2.28)-(2.46) which was given by the Faddeev-Popov approach.

In the end, we note that the effective action and the generating functional obtained in this section, as easily proven, are invariant under a kind of BRST- transformations[21]. The BRST-transformations include the gauge transformations shown in Eqs.(2.13)-(2.16) and the following transformations for ghost fields

$$\delta\overline{C}^i = \frac{\lambda}{\alpha}\partial^\mu A_\mu^i \tag{2.54}$$

$$\delta C^i = -\frac{\lambda}{2}g\varepsilon^{ijk}C^j C^k \tag{2.55}$$

where λ is an infinitesimal anticommuting number. Correspondingly, the group parameters in Eqs.(2.13)-(2.16) should be represented by $\theta^i = \lambda C^i$. The BRST-invariance will leads to a set of Ward-Takahashi identites[20] satisfied by the generating functionals as exhibited in the next section.

3. WARD-TAKAHASHI IDENTITY

In the preceding section, it was mentioned that the generating functional $Z[J]$ is invariant with respect to the BRST-transformations. the BRST-transformations may be written as

$$\delta\Phi_i = \lambda\triangle\Phi_i \tag{3.1}$$

where the $\triangle\Phi_i$ for every field can be explicitly written out from Eqs.(2.13)-(2.16), (2.54) and (2.55). They are shown in the following

$$
\begin{aligned}
&\triangle l = i\{\tfrac{g}{2\sqrt{2}}C^-(1-\gamma_5)\nu - e[C_\gamma + \tfrac{1}{2\sin 2\theta_w}(1-\gamma_5 - 4\sin^2\theta_w)C_z]l\}\\
&\triangle\bar{l} = -i\{\tfrac{g}{2\sqrt{2}}C^+\overline{\nu}(1+\gamma_5) + e\bar{l}[C_\gamma + \tfrac{1}{2\sin 2\theta_w}(1+\gamma_5 - 4\sin^2\theta_w)C_z]\}\\
&\triangle\nu_L = i\tfrac{g}{2}[\tfrac{1}{\cos\theta_w}C_z\nu_L + \sqrt{2}C^+ l_L]\\
&\triangle\overline{\nu}_L = -i[\tfrac{1}{\cos\theta_w}C_z\overline{\nu}_L + \sqrt{2}C^-\bar{l}_L]\\
&\triangle W_\mu^\pm = \partial_\mu C^\pm \mp igC^\pm(\cos\theta_w Z_\mu + \sin\theta_w A_\mu) \pm igW_\mu^\pm(\cos\theta_w C_z + \sin\theta_w C_\gamma)\\
&\triangle Z_\mu = \partial_\mu C_z - ig\cos\theta_w(C^- W_\mu^+ - C^+ W_\mu^-)\\
&\triangle A_\mu = \partial_\mu C_\gamma - ig\sin\theta_w(C^- W_\mu^+ - C^+ W_\mu^-)\\
&\triangle\overline{C^\pm} = \tfrac{1}{\alpha}\partial^\mu W_\mu^\pm\\
&\triangle\overline{C}_z = \tfrac{1}{\alpha}\partial^\mu Z_\mu\\
&\triangle\overline{C}_\gamma = \tfrac{1}{\alpha}\partial^\mu A_\mu\\
&\triangle C^\pm = \pm i\sqrt{2}g(\cos\theta_w C_z + \sin\theta_w C_\gamma)C^\pm\\
&\triangle C_z = ig\cos\theta_w C^+ C^-\\
&\triangle C_\gamma = ig\sin\theta_w C^+ C^-\\
&\triangle\varphi^\pm = \pm\tfrac{i}{2}gC^\pm(H+v)\\
&\triangle\varphi^0 = -\tfrac{g}{2\cos\theta_w}C_z(H+v)\\
&\triangle H = 0
\end{aligned} \tag{3.2}
$$

The last three expressions which come from Eq.(2.13) indicate that in the unitary gauge formulation of the theory, the $SU(2)\times U(1)$ gauge transformation keeps the Higgs field to be invariant, while, creates three Goldstone-type composite fields which consist of the Higgs field and the ghost fields only without concerning Goldstone fields denoted in Eq.(1.5). It is easy to prove that except for $\triangle\overline{C^\pm}, \triangle\overline{C}_z$ and $\triangle\overline{C}_\gamma$, the other functions $\triangle\widetilde{\Phi}_i$ in Eq.(3.2) are nilpotent, $\delta\triangle\widetilde{\Phi}_i = 0$ which means BRST-invariance of the functions $\triangle\widetilde{\Phi}_i$.

Let us define a generalized generating functional by including external sources for the nilpotent functions $\triangle\widetilde{\Phi}_i$

$$Z[J,K] = \frac{1}{N}\int D(\Phi)\exp\{iS_{eff} + i\int d^4x[J_i(x)\Phi_i(x) + K_i(x)\triangle\widetilde{\Phi}_i(x)]\} \tag{3.3}$$

where $J_i\Phi_i$ was shown in Eq.(2.46) and

$$
\begin{aligned}
K_i\triangle\widetilde{\Phi}_i &= u_\mu^+\triangle W^{-\mu} + u_\mu^-\triangle W^{+\mu} + u_z^\mu\triangle Z_\mu + u_\gamma^\mu\triangle A_\mu + v^+\triangle C^- + v^-\triangle C^+ + v_z\triangle C_z\\
&+v_\gamma\triangle C_\gamma + \overline{\chi}_l\triangle l + \triangle\bar{l}\chi_l + \overline{\chi}_\nu\triangle\nu_L + \triangle\overline{\nu}_L\chi_\nu + K^+\triangle\varphi^- + K^-\triangle\varphi^+ + \triangle\varphi^0 K^0
\end{aligned} \tag{3.4}
$$

On making the BRST-transformation to the functional $Z[J,K]$ and noticing the BRST-invariance of the functional, we obtain a W-T identity such that[12,21,22]

$$\frac{1}{N}\int D(\Phi)\int d^4x(\pm)J_i(x)\triangle\Phi_i(x)\exp\{iS_{eff} + iJ\cdot\Phi + K\cdot\triangle\widetilde{\Phi}\} = 0 \tag{3.5}$$

where the signs "+" and "−" attribute to commuting and anticommuting sources J_i respectively. The above identity may be represented in terms of differentials of $Z[J,K]$ with respect to the external sources. Here we only write down specifically the identity satisfied by the generating functional of connected Green's functions $W[J,K]$ which is defined by $Z = exp(iW)$[19,21,22]

$$
\begin{aligned}
&\int d^4x\{\xi_l(x)\tfrac{\delta}{\delta\chi_l(x)} - \overline{\xi}_l(x)\tfrac{\delta}{\delta\overline{\chi}_l(x)} + \xi_\nu(x)\tfrac{\delta}{\delta\chi_\nu(x)} - \overline{\xi}_\nu(x)\tfrac{\delta}{\delta\overline{\chi}_\nu(x)} + J_\mu^+(x)\tfrac{\delta}{\delta u_\mu^+(x)} + J_\mu^-(x)\tfrac{\delta}{\delta u_\mu^-(x)}\\
&+J_z^\mu(x)\tfrac{\delta}{\delta u_z^\mu(x)} + J_\gamma^\mu(x)\tfrac{\delta}{\delta u_\gamma^\mu(x)} - \overline{\eta}^+(x)\tfrac{\delta}{\delta v^+(x)} - \overline{\eta}^-(x)\tfrac{\delta}{\delta v^-(x)} - \overline{\eta}_z(x)\tfrac{\delta}{\delta v_z(x)} - \overline{\eta}_\gamma(x)\tfrac{\delta}{\delta v_\gamma(x)}\\
&+\tfrac{1}{\alpha}\partial_\mu^x\tfrac{\delta}{\delta J_\mu^+(x)}\eta^+(x) + \tfrac{1}{\alpha}\partial_\mu^x\tfrac{\delta}{\delta J_\mu^-(x)}\eta^-(x) + \tfrac{1}{\alpha}\partial_x^\mu\tfrac{\delta}{\delta J_z^\mu(x)}\eta_z(x) + \tfrac{1}{\alpha}\partial_x^\mu\tfrac{\delta}{\delta J_\gamma^\mu(x)}\eta_\gamma(x)\}W[J,K]\\
&= 0
\end{aligned} \tag{3.6}
$$

When we make a translation transformation: $\overline{C}^i \to \overline{C}^i + \overline{\lambda}^i$ to the functional $Z[J, K]$, then differentiate the functional with respect to the $\overline{\lambda}^i(x)$ and finally set $\overline{\lambda}^i = 0$, we get such a ghost equation that[20-22]

$$\frac{1}{N} \int D(\Phi)[\eta^i(x) + \triangle F^i(x)] \exp\{\{iS_{eff} + iJ \cdot \Phi + K \cdot \triangle \widetilde{\Phi}\} = 0 \tag{3.7}$$

where

$$\triangle F^i(x) = \partial_x^\mu [D_\mu^{ij}(x) C^j(x)] + \frac{i}{8} \alpha g^i g^j [V^+ \tau^i \tau^j \phi_0(x) + \phi_0^+(x) \tau^j \tau^i V] C^j(x) \tag{3.8}$$

From the above equation, we may write out the following ghost equations via the functional $W[J, K]$

$$\eta^+(x) + \partial_\mu^x \frac{\delta W}{\delta u_\mu^-(x)} - i\alpha M_w \frac{\delta W}{\delta K^-(x)} = 0 \tag{3.9}$$

$$\eta^-(x) + \partial_\mu^x \frac{\delta W}{\delta u_\mu^+(x)} + i\alpha M_w \frac{\delta W}{\delta K^+(x)} = 0 \tag{3.10}$$

$$\eta_z(x) + \partial_x^\mu \frac{\delta W}{\delta u_z^\mu(x)} + \alpha M_z \frac{\delta W}{\delta K^0(x)} = 0 \tag{3.11}$$

$$\eta_\gamma(x) + \partial_x^\mu \frac{\delta W}{\delta u_\gamma^\mu(x)} = 0 \tag{3.12}$$

With introduction of the generating functional of proper vertices defined by

$$\Gamma[\Phi, K] = W[J, K] - \int d^4 x J_i(x) \Phi_i(x) \tag{3.13}$$

where Φ_i are the vacuum expectation values of the field operators in the presence of external sources, one may easily write down the representations of the identity in Eq.(3.6) and the ghost equations in Eqs.(3.9)-(3.12) through the functional Γ which we do not list here.

4. INCLUSION OF QUARKS

In the previous sections, the quantum electroweak theory for leptons has been built up starting from the Lagrangian given in the unitary gauge. For completeness, in this section, the corresponding theory for quarks will be briefly formulated. The $SU(2) \times U(1)$ symmetric Lagrangian describing the interactions of quarks with the gauge bosons and the Higgs particle is, in the unitary gauge, of the form[23,24]

$$\begin{aligned}
\mathcal{L}_q &= \overline{Q}_{jL} i\gamma^\mu D_\mu Q_{jL} + \overline{U}_{jR} i\gamma^\mu D_\mu U_{jR} + \overline{D}_{jR}^\theta i\gamma^\mu D_\mu D_{jR}^\theta \\
&- \frac{1}{\sqrt{2}} f_j(U)[\overline{Q}_{jL}\widetilde{\phi}_0 U_{jR} + \overline{U}_{jR}\widetilde{\phi}_0^+ Q_{jL}] \\
&- \frac{1}{\sqrt{2}} f_j(D)[\overline{Q}_{jL}\phi_0 D_{jR}^\theta + \overline{D}_{jR}^\theta \phi_0^+ Q_{jL}]
\end{aligned} \tag{4.1}$$

where the repeated index j ($j = 1, 2, 3$) which is the label of quark generation implies summation,

$$Q_{jL} = \begin{pmatrix} U_{jL} \\ D_{jL}^\theta \end{pmatrix} \tag{4.2}$$

is the $SU(2)$ doublet (for a given j) constructed by the left-handed quarks in which U_j stands for the up-quark u, c or t and D_j^θ is defined by

$$D_j^\theta = V_{jk} D_k, \tag{4.3}$$

10

here V_{jk} denote the elements of the unitary $K-M$ mixing matrix V[24,25] and D_k symbolizes the down-quark d,s or b, U_{jR} and D_{jR} designate the SU(2) singlets for the right-handed up-quarks and down-quarks respectively, ϕ_0 is the scalar field doublet defined in Eq.(2.9), $\widetilde{\phi}_0$ is the charge-conjugate of ϕ_0 which is defined by[24]

$$\widetilde{\phi}_0 = i\tau_2\phi_0^* = \left(\begin{array}{c} H+v \\ 0 \end{array} \right) \tag{4.4}$$

$f_j(U)$ and $f_j(D)$ are the coupling constants.

In Eq.(4.1), the first three terms are responsible for determining the kinetic energy terms of quarks and the interactions between the quarks and the gauge bosons, and the remaining terms which are simpler than those chosen in the R_α-gauge theory are designed to yield the quark masses and the couplings between the quarks and the Higgs particle. By using the expressions shown in Eqs.(4.2)-(4.4) and their conjugate ones as well as the following eigen-equations

$$YQ_{jL} = \frac{1}{3}Q_{jL} \ , \ YU_{jR} = \frac{4}{3}U_{jR} \ , \ YD_{jR} = -\frac{2}{3}D_{jR} \ ,$$
$$\tau^a U_{jR} = 0 \ , \ \tau^a D_{jR} = 0 \tag{4.5}$$

the Lagrangian in Eq.(4.1) will be represented as

$$\mathcal{L}_q = \overline{U}_j[i\gamma^\mu\partial_\mu - m_j(U)]U_j + \overline{D}_j[i\gamma^\mu\partial_\mu - m_j(D)]D_j$$
$$+ \frac{g}{\sqrt{2}}[\widetilde{j}_{L\mu}^- W^{+\mu} + \widetilde{j}_{L\mu}^+ W^{-\mu}] + e\widetilde{j}_\mu^{em}A^\mu + \frac{e}{\sin 2\theta_w}\widetilde{j}_\mu^0 Z^\mu$$
$$- \frac{1}{\sqrt{2}}f_j(U)\overline{U}_jU_jH - \frac{1}{\sqrt{2}}f_j(D)\overline{D}_jD_jH \tag{4.6}$$

where

$$\widetilde{j}_{L\mu}^- = \overline{U}_{jL}\gamma_\mu V_{jk}D_{kL} = (\widetilde{j}_{L\mu}^+)^+ \tag{4.7}$$

$$\widetilde{j}_\mu^{em} = \frac{2}{3}\overline{U}_j\gamma_\mu U_j - \frac{1}{3}\overline{D}_j i\gamma_\mu D_j \tag{4.8}$$

$$\widetilde{j}_\mu^0 = \overline{U}_j\gamma_\mu\frac{1}{2}(1-\gamma_5)U_j - \overline{D}_j\gamma_\mu\frac{1}{2}(1-\gamma_5)D_j - 2\sin^2\theta_w\widetilde{j}_\mu^{em} \tag{4.9}$$

and

$$m_j(U) = \frac{1}{\sqrt{2}}f_j(U)v, m_j(D) = \frac{1}{\sqrt{2}}f_j(D)v \tag{4.10}$$

From the procedure of quantization as stated in Sect.2, it is clear to see that the Lagrangian in Eq.(4.1) or (4.6), as a part of the total Lagrangian of the lepton-quark system, may simply be added to the effective Lagrangian denoted in Eq.(2.34).

5. THE THEORY WITHOUT HIGGS BOSON

In the previous sections, we described the theory given in the unitary gauge which includes the Higgs boson in it. Whether the Higgs particle really exists in the world or not nowadays becomes a central problem in particle physics. If the Higgs particle could not be found in experiment, one may ask whether this particle can be thrown out from the electroweak theory without breaking the original gauge-symmetry of the theory? Recall that the gauge transformation does not alter the vector nature of the gauge boson fields and the spinor character of the fermion fields. They all remain the same numbers of components before and after gauge transformations. But, the situation for the scalar field is different. In the functional space spanned by the four scalar functions, the scalar function ϕ defined in Eq.(1.4) forms a four-dimensional vector and the gauge transformations act as rotations. A special rotation (the so-called Higgs transformation) can convert a four-dimensional vector ϕ to the one which has only one nonvanishing component along the Higgs direction, i.e. the function $H(x)+v$. But, any rotation does not change the magnitude of the vector ϕ, $|\phi| = H+v$. In the Higgs mechanism, although the vacuum state can be chosen in different ways, it is usually chosen in the Higgs direction. This choice is made just from the physical requirement. The scalar field function $H(x)+v$ was viewed as physical. But, the theory does not tell us what the vacuum expectation value v should be. It may be very

large or very small. The scalar fields and their vacuum expectation value were introduced originally for giving some gauge bosons and fermions masses . For this purpose, we may simply limit ourself to require the magnitude of the vector ϕ to be equal to the vacuum expectation v, $|\phi| = v$. In this case, we may set $\phi \to V$ where V was represented in Eq.(1.6). The vacuum V generally is a function of space-time; but in practice, as usual, it is chosen to be a constant along the Higgs direction. With this choice, the Lagrangian of the system under consideration is still represented by Eq.(2.1) except that the Lagrangian $\mathcal{L}_\phi$ in Eq.(2.8) is reduced to

$$\mathcal{L}_\phi = \mathcal{L}_{GM} + \mathcal{L}_{lm} \tag{5.1}$$

$$\mathcal{L}_{GM} = (D^\mu V)^+(D_\mu V) - \mu^2 V^+ V - \lambda(V^+ V)^2$$
$$= \tfrac{1}{2} F_\mu^+ F^\mu \tag{5.2}$$

where

$$F_\mu = \frac{1}{2}(g\tau^a A_\mu^a + g' B_\mu)V \tag{5.3}$$

and

$$\mathcal{L}_{lm} = -f_l(\overline{L}V l_R + \overline{l}_R V^+ L) \tag{5.4}$$

The $\mathcal{L}_{GM}$ and $\mathcal{L}_{lm}$ now only play the role of generating the mass terms of $\mathrm{W}^\pm$ and Z^0 bosons and charged fermions respectively. Correspondingly, the gauge transformation in Eq.(2.13) is reduced to

$$\delta V = \frac{i}{2}(g\tau^a \theta^a + g'\theta^0)V \tag{5.5}$$

here the gauge transformation of V , as pointed out before, is chosen to be the same as for the scalar function ϕ. The other gauge transformations are still represented in Eqs.(2.14)-(2.16). It is easy to see that the Lagrangian $\mathcal{L}_{lm}$ is gauge-invariant, $\delta\mathcal{L}_{lm} = 0$. By the gauge transformations written in Eqs.(5.6) and (2.14)-(2.16), one may find

$$\delta F_\mu = \tfrac{i}{2}(g\tau^a \theta^a + g'\theta^0)F_\mu + \partial_\mu(g\tau^a \theta^a + g'\theta^0)V$$
$$\delta F_\mu^+ = -\tfrac{i}{2}F_\mu^+(g\tau^a \theta^a + g'\theta^0) + V^+\partial_\mu(g\tau^a \theta^a + g'\theta^0) \tag{5.6}$$

With these transformations, it can be proved that the action given by the Lagrangian $\mathcal{L}_{GM}$ and hence the action given by the total Lagrangian $\mathcal{L}$ is gauge-invariant under the Lorentz condition written in Eqs.(1.8) and (1.9),

$$\delta S = \int d^4 x \delta\mathcal{L} = \int d^4 x \delta\mathcal{L}_{GM} = \tfrac{1}{2}\int d^4 x(\delta F_\mu^+ F^\mu + F_\mu^+ \delta F^\mu)$$
$$= -\frac{v^2}{4}\int d^4 x\{g^2(\partial^\mu A_\mu^1 \theta^1 + \partial^\mu A_\mu^2 \theta^2) + (g'\theta^0 - g\theta^3)(g'\partial^\mu B_\mu - g\partial^\mu A_\mu^3)\}$$
$$= 0 \tag{5.7}$$

As emphasized in Ref.[1] and in Introduction, for a constrained system such as the massive gauge field, whether a system is gauge-invariant or not should be seen from that whether the action of the system, which is given in the physical subspace defined by the introduced constraint conditions, is gauge-invariant or not. Thus, according to Eq.(5,7), the Lagrangian given in Eq.(2.1) with the $\mathcal{L}_\phi$ given in Eqs.(5.1)-(5.4) still ensures the theory to have the $SU(2) \times U(1)$ gauge-symmetry.

Now, let us to quantize the theory by means of the Lagrange multiplier method. For this purpose, the Lorentz conditions in Eq. (2.12) are extended to

$$\partial^\mu A_\mu^i + \alpha\lambda^i = 0 \tag{5.8}$$

where $i = 0, 1, 2, 3$ (Note : the conditions In Eqs.(2.10) and (2.11) are not necessarily considered here because they become trivial identities in this case). When these conditions are incorporated into the Lagrangian by the Lagrange multiplier method, we have

$$\mathcal{L}_\lambda = \mathcal{L} + \lambda^i \partial^\mu A_\mu^i + \frac{1}{2}\alpha(\lambda^i)^2 \tag{5.9}$$

where $\mathcal{L}$ was written in Eqs.(2.1)-(2.7) and (5.1)-(5.4). As mentioned before, in order to build a gauge-invariant theory, the action given by the above Lagrangian must be required to be gauge-invariant. By making use of the gauge transformation in Eq.(2.16) and the gauge transformation of the Lagrangian $\mathcal{L}$ which may be read from Eq.(5.7),

$$\delta\mathcal{L} = -\frac{v^2}{4}\{g^2(\partial^\mu A_\mu^1 \theta^1 + \partial^\mu A_\mu^2 \theta^2) + (g'\theta^0 - g\theta^3)(g'\partial^\mu B_\mu - g\partial^\mu A_\mu^3)\}$$
$$= -m_W^2 \partial^\mu W_\mu^+ \theta^- - m_W^2 \partial^\mu W_\mu^- \theta^+ - m_Z^2 \partial^\mu Z_\mu \theta^0 \tag{5.10}$$

where we have used the definitions denoted in Eqs.(2.29)-(2.33) and (2.37) and utilizing the constraint condition in Eq.(5.8), one can derive

$$\delta S = \int d^4x \{\delta\mathcal{L} - \frac{1}{\alpha}\partial^\mu A_\mu^i \partial^\nu \delta A_\nu^i\}$$
$$= -\frac{1}{\alpha}\{\partial^\nu W_\nu^+(\partial^\mu \delta W_\mu^- + \alpha m_W^2 \theta^-) + \partial^\nu W_\nu^-(\partial^\mu \delta W_\mu^+ + \alpha m_W^2 \theta^+)$$
$$-\partial^\nu Z_\nu(\partial^\mu \delta Z_\mu + \alpha m_Z^2 \theta^0) - \partial^\nu A_\nu \partial^\mu \delta A_\mu\}$$
$$= 0 \tag{5.11}$$

According to Eq.(5.8), $\frac{1}{\alpha}\partial^\nu W_\nu^\pm \neq 0, \frac{1}{\alpha}\partial^\nu Z_\nu \neq 0$ and $\frac{1}{\alpha}\partial^\nu A_\nu \neq 0$, therefore, we may obtain from Eq.(5.11) the constraint equations on the gauge group

$$\partial^\mu \delta W_\mu^\pm + \alpha m_W^2 \theta^\pm = 0 \tag{5.12}$$

$$\partial^\mu \delta Z_\mu + \alpha m_Z^2 \theta^0 = 0 \tag{5.13}$$

$$\partial^\mu \delta A_\mu = 0 \tag{5.14}$$

Defining the ghost field functions C^i by $\theta^i = \xi C^i$ and omitting the infinitesimal anticommuting number ξ from Eqs.(5.12)-(5.14), we get the ghost equations as follows

$$\partial^\mu \Delta W_\mu^\pm + \alpha m_W^2 C^\pm = 0 \tag{5.15}$$

$$\partial^\mu \Delta Z_\mu + \alpha m_Z^2 C^0 = 0 \tag{5.16}$$

$$\partial^\mu \Delta A_\mu = 0 \tag{5.17}$$

where $\Delta W^\pm, \Delta Z$ and ΔA_μ were defined in Eq.(3.2). The constraint equations in Eqs.(5.15)-(5.17) may also be incorporated into the Lagrangian in Eq.(5.9) by the Lagrange undetermined multiplier method to give a generalized Lagrangian

$$\mathcal{L}_\lambda^* = \mathcal{L} + \lambda^i \partial^\mu A_\mu^i + \frac{1}{2}\alpha(\lambda^i)^2 + \mathcal{L}_{gh} \tag{5.18}$$

where

$$\mathcal{L}_{gh} = \overline{C}^+(\partial^\mu \Delta W_\mu^- + \alpha m_W^2 C^-) + \overline{C}^-(\partial^\mu \Delta W_\mu^+ + \alpha m_W^2 C^+) + \overline{C}_Z(\partial^\mu \Delta Z_\mu + \alpha m_Z^2 C^0) + \overline{C}_\gamma \partial^\mu \Delta A_\mu$$
$$= \overline{C}^-(\Box + \alpha M_w^2)C^+ + \overline{C}^+(\Box + \alpha M_w^2)C^- + \overline{C}_z(\Box + \alpha M_z^2)C_z + \overline{C}_\gamma \Box C\gamma - ig\{(\partial^\mu \overline{C}^+ C^-$$
$$-\partial^\mu \overline{C}^- C^+)(\cos\theta_w Z_\mu + \sin\theta_w A_\mu) + (\partial^\mu \overline{C}^- W_\mu^+ - \partial^\mu \overline{C}^+ W_\mu^-)(\cos\theta_w C_z + \sin\theta_w C_\gamma)$$
$$+(\cos\theta_w \partial^\mu \overline{C}_z + \sin\theta_w \partial^\mu \overline{C}_\gamma)(C^+ W_\mu^- - C^- W_\mu^+)\} \tag{5.19}$$

which is simpler than that given in Eq.(2.45).

Completely following the procedure shown in Eq.(2.53), we may use the Lagrangian in Eq.(5.18) to construct the generating functional of Green's functions which is formally as the same as that in Eq.(2.53). After calculating the integral over λ^i, we obtain an effective Lagrangian like this

$$\mathcal{L}_{eff} = \mathcal{L}_G + \mathcal{L}_F + +\mathcal{L}_{gf} + \mathcal{L}_{gh} + \mathcal{L}_q \tag{5.20}$$

where $\mathcal{L}_G, \mathcal{L}_F$ and $\mathcal{L}_{gf}$ were represented in Eqs.(2.34)-(2.37), (2.38)-(2.42) and (2.44) respectively, while, $\mathcal{L}_{gh}$ is now given in Eq.(5.19) and $\mathcal{L}_q$ is still described in section 4 except that the scalar fields ϕ_0 and $\widetilde{\phi}_0$ in Eq.(4.1) are now replaced by the vacuum V denoted in Eq.(1.6) and, therefore, the last two terms in Eq.(4.6) are absent in the present case. The results stated above can equally be derived by employing the Faddeev-Popov approach. The Lagrangian given in this section establishes the quantum electroweak theory without involving the Higgs boson. This theory may actually be written out from the theory described in sections 2, 3 and 4 by dropping out all the terms related to the Higgs boson.

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6. REMARKS

The quantum electroweak theory described in the previous sections is not only simpler than the ordinary R_α-gauge theory, but also would safely ensure the theory to be renormalizable due to the absence of the Goldstone bosons. To this end, we would like to mention the role played by the Goldstone bosons in the ordinary theory. As illustrated by the example presented in Appendix A which shows the tree diagrams of antineutrino-electron scattering and their S-matrix elements, the Goldstone boson propagator in Fig.(b) just plays the role of cancelling out the contribution arising from the unphysical part of the gauge boson propagator in Fig.(a) to the S-matrix element. Therefore, the ordinary R_α-gauge theory can naturally guarantee the tree unitarity of the S-matrix element. However, considering that the both diagrams in Figs.(a) and (b), as subgraphs, will appear, companying each other, in higher order Feynman diagrams and they can be replaced by the only one diagram shown in Fig.(a) in which the gauge boson propagator is given in the unitary gauge, the bad ultraviolet divergence of the term in the latter propagator would cause some difficulties of renormalization as indicated in Ref.[16]. In contrast, in the theory presented in this paper, there are not the Feynman diagrams involving the Goldstone bosons like Fig.(b), therefore, the aforementioned term of bad ultraviolet behavior does not appear in the massive gauge boson propagator and any Feynman integrals to spoil the renormalizability of the theory. However, the present theory formulated in the α-gauge will not content with the unitarity condition of S-matrix elements due to the presence of the axial current and the mass difference between the charged particle and neutral one. How to understand and resolve this problem? As explained in Appendix B, the propagator in Eq.(1.1) is given by the physical transverse vector potential which is on the mass-shell (this point is clearly seen in the canonical quantization; but not so clearly in the path-integral quantization), While, the propagator in Eq.(1.6) is given by the full vector potential which contains an unphysical longitudinal component in it and therefore is off-mass-shell. This propagator is suitable to be used for calculating Green's functions which are off-shell. In the limit: $\alpha \to \infty$, the α-gauge propagator is converted to the unitary gauge one since the unphysical part of the former propagator vanishes in the limit. Therefore, the propagator given in the α-gauge can be considered as a kind of parametrization (or say, regularization) of the propagator given in the unitary gauge, somehow similar to the regularization procedure in the renormalization scheme. In view of this point of view, we have no reasons to require the α-gauge theory to directly give the on-shell S-matrix elements. Nevertheless, due to its renormalizable character, it is suitable to use such a theory at first in practical calculations of the S-matrix elements and then the α-limiting procedure mentioned above is necessary to be required in the final step of the calculations[18]. It is noted that the α-limiting procedure can only be applied to the massive gauge boson propagators. This means that we have to make distinction between the gauge parameters appearing in the massive gauge boson propagators and the photon propagator in the procedure. Certainly, by the limiting procedure, the unitarity of the theory is always ensured in spite of whether the currents involved in the theory are conserved or not. At last, we mention that the ordinary R_α-gauge theory, actually, can also be viewed as another kind of parametrization of the unitary gauge theory because in the limit: $\alpha \to \infty$, the theory in the R_α-gauge directly goes over to the one in the unitary gauge. The question arises: which parametrization is suitable? The answer should be given by the requirement that which theory allows us to perform the renormalization safely and give correct physical results. An essential point to fulfil this requirement is that the theory must maintain the original gauge-symmetry, just as the same requirement for the regularization procedure of renormalization. The α-gauge theory formulated in this paper is exactly of the $SU(2) \times U(1)$ gauge symmetry. As shown in Sect.3, this gauge symmetry is embodied in the W-T identities satisfied by the generating functionals. From these W-T identities, one may readily derive a set of W-T identities obeyed by Green's functions and vertices which establish correct relations between the Green's functions and the vertices and provide a firm basis for performing the renormalization of the theory. These subjects will be discussed in the subsequent papers.

7. ACKNOWLEDGMENT

8. APPENDIX A: THE LOWEST-ORDER S-MATRIX ELEMENT OF ANTINEUTRINO-ELECTRON SCATTERING

The tree diagrams representing the antineutrino-electron scattering are shown in Figs.(a)- (c) in which the internal lines are respectively the W-boson propagator, the Goldstone boson propagator and the Z^0 boson propagator. According to the Feynman rules given in the R_α-gauge theory[7,23], the corresponding S-matrix element can be written as

$$T_{fi} = T_w + T_G + T_Z \tag{A.1}$$

where T_w, T_G and T_Z represent respectively the S-matrix elements of Figs.(a), (b) and (c),

$$T_w = \left(\frac{-ig}{2\sqrt{2}}\right)^2 \overline{u}_e(q_1)\gamma^\mu(1-\gamma_5)v_\nu(q_2)\overline{v}_\nu(p_2)\gamma^\nu(1-\gamma_5)u_e(p_1)$$
$$\times \frac{-i}{k^2-M_w^2+i\varepsilon}[g_{\mu\nu} - (1-\alpha)\frac{k_\mu k_\nu}{k^2-\alpha M_w^2}] \tag{A.2}$$

$$T_G = \left(\frac{if_e}{2}\right)^2 \overline{u}_e(q_1)(1-\gamma_5)v_\nu(q_2)\overline{v}_\nu(p_2)(1+\gamma_5)u_e(p_1)\frac{i}{k^2-\alpha M_w^2} \tag{A.3}$$

where $k = p_1 + p_2 = q_1 + q_2$ and the subscripts "e" and "ν" mark which particles the spinors belong to, and

$$T_Z = \frac{g^2}{8\cos^2\theta_w}\overline{u}_e(q_1)\gamma^\mu[\tfrac{1}{2}(1-\gamma_5) - 2\sin^2\theta_w]u_e(p_1)\overline{v}_\nu(p_2)\gamma^\nu(1-\gamma_5)v_\nu(q_2)$$
$$\times \frac{-i}{k^2-M_Z^2+i\varepsilon}[g_{\mu\nu} - (1-\alpha)\frac{k_\mu k_\nu}{k^2-\alpha M_Z^2}] \tag{A.4}$$

where $k = q_1 - p_1 = p_2 - q_2$.

Noticing the decomposition

$$g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2-\alpha M_j^2} = g_{\mu\nu} - \frac{k_\mu k_\nu}{M_j^2} + \frac{(k^2-M_j^2)k_\mu k_\nu}{(k^2-\alpha M_j^2)M_j^2} \tag{A.5}$$

where $j = W$ or Z, the matrix element in Eq.(A.2) can be rewritten as

$$T_w = T_w^{(1)} + T_w^{(2)} \tag{A.6}$$

where

$$T_w^{(1)} = \left(\frac{-ig}{2\sqrt{2}}\right)^2 \overline{u}_e(q_1)\gamma^\mu(1-\gamma_5)v_\nu(q_2)\overline{v}_\nu(p_2)\gamma^\nu(1-\gamma_5)u_e(p_1)\frac{-i}{k^2-M_w^2}(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_w^2}) \tag{A.7}$$

and

$$T_w^{(2)} = \left(\frac{-ig}{2\sqrt{2}}\right)^2 \overline{u}_e(q_1)\gamma^\mu(1-\gamma_5)v_\nu(q_2)\overline{v}_\nu(p_2)\gamma^\nu(1-\gamma_5)u_e(p_1)\frac{-ik_\mu k_\nu}{M_w^2(k^2-\alpha M_w^2)} \tag{A.8}$$

Applying the energy-momentum conservation and the Dirac equation, it is easy to see

$$\overline{u}_e(q_1)\gamma^\mu k_\mu(1-\gamma_5)v_\nu(q_2)\overline{v}_\nu(p_2)\gamma^\nu k_\nu(1-\gamma_5)u_e(p_1)$$
$$= m_e^2\overline{u}_e(q_1)(1-\gamma_5)v_\nu(q_2)\overline{v}_\nu(p_2)(1+\gamma_5)u_e(p_1) \tag{A.9}$$

Inserting Eq.(A.9) into Eq.(A8) and using the relation

$$f_e = \frac{gm_e}{\sqrt{2}M_w} \tag{A.10}$$

we find

$$T_w^{(2)} = -T_G \tag{A.11}$$

Therefore, we have

$$T_w + T_G = T_w^{(1)} \tag{A.12}$$

This result shows us that the contributions from Figs.(a) and (b) is equal to the contribution from Fig.(a) provided that the W-boson propagator in Fig.(a) is replaced by the one given in the unitary gauge. Similarly, employing Eq.(A.5) and the Dirac equations for electron and neutrino, Eq.(A.4) becomes

$$T_Z = \frac{g^2}{8\cos^2\theta_w}\overline{u}_e(q_1)\gamma^\mu[\tfrac{1}{2}(1-\gamma_5) - 2\sin^2\theta_w]u_e(p_1)\overline{v}_\nu(p_2)\gamma^\nu(1-\gamma_5)v_\nu(q_2)$$
$$\times \frac{-i}{k^2-M_Z^2+i\varepsilon}[g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2-M_Z^2}] \tag{A.13}$$

where the Z^0 boson propagator is also given in the unitary gauge.

9. APPENDIX B: ON THE MASSIVE GAUGE BOSON PROPAGATORS

To help understanding of the nature of the massive gauge boson propagators given in the unitary gauge and the α-gauge, we show how these propagators are derived in the formalism of canonical quantization. For simplicity, we only take the Lagrangian of a free massive vector field[22]

$$\mathcal{L}_0 = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m^2 V^\mu V_\mu \tag{B.1}$$

where V_μ is the vector potential for a massive vector field ($W^\pm$ or Z^0). To give a complete formulation of the field dynamics, the above Lagrangian must be constrained by the Lorentz condition: $\partial^\mu V_\mu = 0$ whose solution is $V_{L\mu} = 0$. Substituting this solution in Eq.(B.1), the Lagrangian will be merely expressed by the transverse vector potential $V_{T\mu}$. Since the $V_{T\mu}$ completely describes the three independent polarization states of the massive vector field, in operator formalism, it can be represented by the following Fourier integral

$$V_{T\mu}(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \sum_{\lambda=1}^{3} \frac{\epsilon_\mu^\lambda(\mathbf{k})}{\sqrt{2\omega(\mathbf{k})}}[a_\lambda(\mathbf{k})e^{-ikx} + a_\lambda^+(\mathbf{k})e^{ikx}] \tag{B.2}$$

where $\omega(\mathbf{k})$ is the energy of free particle and $\epsilon_\mu^\lambda(\mathbf{k})$ is the unit vector of polarization satisfying the transversity condition: $k^\mu \epsilon_\mu^\lambda(\mathbf{k}) = 0$, which corresponds to the transversity condition: $\partial^\mu V_{T\mu}(x) = 0$. By using the familiar canonical commutation relations between the annihilation operator $a_\lambda(\mathbf{k})$ and the creation one $a_\lambda^+(\mathbf{k})$, as derived in the literature[22], one gets the propagator for the transverse vector potential as follows

$$iD_{\mu\nu}(x-y) = \langle 0|T\{V_{T\mu}(x)V_{T\nu}(y)\}|0\rangle = \int \frac{d^4k}{(2\pi)^4}iD_{\mu\nu}(k)e^{-ik(x-y)} \tag{B.3}$$

where $iD_{\mu\nu}(k)$ is just the one shown in Eq.(1.1) here a non-covariant part of the propagator has been omitted because it will be cancelled in S-matrix elements by the non-covariant term in the interaction Hamiltonian .

On the other hand, when the Lorentz condition is generalized to the from: $\partial^\mu V_\mu + \alpha\lambda = 0$, where λ acts as a Lagrange multiplier, and incorporated into the Lagrangian by the Lagrangian multiplier method, one may obtain the Stückelberg's Lagrangian[22]

$$\mathcal{L}_0 = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m^2 V^\mu V_\mu - \frac{1}{2\alpha}(\partial^\mu V_\mu)^2 \tag{B.4}$$

According to the spirit of the Lagrange multiplier method, every component of the vector potential in the above Lagrangian can be treated as independent. From the Lagrangian, one may derive the equation of motion

$$\left(\Box + m^2\right)V_\mu - (1 - \frac{1}{\alpha})\partial_\mu\partial_\nu V^\nu = 0 \tag{B.5}$$

Taking divergence of the above equation leads to a scalar field equation

$$(\Box + \mu^2)\varphi = 0 \tag{B.6}$$

where $\varphi = \partial^\mu V_\mu$ and $\mu^2 = \alpha m^2$. Now the full vector potential can be expressed as

$$V_\mu = V_{T\mu} + V_{L\mu} \tag{B.7}$$

where $V_{T\mu}$ is the transverse part of the potential which was represented in Eq.(B.2) and $V_{L\mu}$ is the longitudinal part of the potential which is defined by $V_{L\mu} = \frac{1}{\mu^2}\partial_\mu\varphi$ and can be expanded as

$$V_{L\mu}(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{\epsilon_\mu^0(k)}{\sqrt{2\varpi}}[a_0(\mathbf{k})e^{-ikx} + a_0^+(\mathbf{k})e^{ikx}] \tag{B.8}$$

where $\epsilon_\mu^0(k) = k_\mu/m$ and ϖ is the energy of the scalar particle of mass μ. With the expressions presented in Eqs.(B.7), (B.2) and (B.8), in the canonical formalism, it is easy to derive the propagator for the full vector potential[22]

$$iD_{\mu\nu}(x-y) = \langle 0|T\{V_\mu(x)V_\nu(y)\}|0\rangle = \int \frac{d^4k}{(2\pi)^4}iD_{\mu\nu}(k)e^{-ik(x-y)} \tag{B.9}$$

where $iD_{\mu\nu}(k)$ is exactly of the form as written in Eq.(1.7). This propagator is usually derived in the path-integral formalism and often used in perturbative calculations. The propagator in Eq.(1.7) contains two parts: The first part which is usually given in the Landau gauge ($\alpha = 0$) is transverse with respect to the off-shell momentum k_μ, while the second part is longitudinal for the off-shell momentum. According to the decomposition in Eq.(A.5), the above propagator can be divided into such two parts: one is that given in the unitary gauge as shown in Eq.(1.1); another is

$$iD_{\mu\nu}(k) = -i\frac{k_\mu k_\nu}{\left(k^2 - \alpha M^2\right)M^2} \tag{B.10}$$

The two parts are respectively transverse and longitudinal with respect to the on-shell momentum k_μ. As one can see, the off-shell transverse propagator (in the Landau gauge) and the on-shell-transverse propagator (in the unitary gauge) are given by different limits: $\alpha \to 0$ and $\alpha \to \infty$, respectively.

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11. FIGURE CAPTIONS

Fig.(a): The tree diagram representing the antineutrino-electron scattering which takes place via the interaction mediated by one W-boson exchange.

Fig.(b): The tree diagram representing the antineutrino-electron scattering which takes place via the interaction mediated by one Goldstone boson exchange.

Fig.(c): The tree diagram representing the antineutrino-electron scattering which takes place via the interaction mediated by one Z^0-boson exchange.